Portfolio Insurance

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Dynamic portfolio insurance

- Static portfolio insurance using puts
  - identify portfolio insurance date T
  - buy puts with maturity T, strike K (garanteed floor)
  - with a one-to-one relationship between the number of risky assets in the portfolio to insure and the number of put options to purchase
- Terminal cash-flows

	$S_T < K$	$S_T > K$
Long stock	S_T	S_T
Long put	$(K - S_T)$	0
Total	K	S_T

Dynamic portfolio insurance

- Practical problems with static portfolio insurance
  - for some indices traded options are only of american style
  - desired strike prices and maturities may not be available or liquid enough
  - the underlying asset used for the options may differ from the portfolio to be insured
  - legal constraints might forbid investing in derivatives

Dynamic portfolio insurance

- Rather than buying an option, dynamic portfolio insurance consists in
  - creating the put option synthetically
    - by taking a position in the underlying asset
    - and adjusting it regularly so that the delta of the position is maintained equal to the delta of the required option
  - the synthetic option can be created from trades in stocks themselves or from trades in futures contracts
- This technique is known as OBPI (Option-Based Portfolio Insurance) and was employed by LOR (Leland –O'Brien – Rubinstein) until the October 1987 market crash (see below)

OBPI – the principle (1)

- According to the Black-Scholes formula, the expression for the value of a put with maturity T is:

$$P(t) = -S(t)N(-d_1) + Ke^{-r(T-t)}N(-d_2) \quad (1)$$

- First term of the RHS : amount invested in underlying asset (short position)
- Second term of the RHS : amount invested in risk-free asset paying r (long position)
- Portfolio $S(t) + P(t)$ will pay at least K on date T (see first slide)

OBPI – the principle (2)

- Using the expression of $P(t)$ in (1), $S(t) + P(t)$ can be rewritten as:

$$\begin{aligned} S(t) + P(t) &= S(t) - S(t)N(-d_1) + Ke^{-r(T-t)}N(-d_2) \\ &= S(t)[1 - N(-d_1)] + Ke^{-r(T-t)}N(-d_2) \\ &= S(t)N(d_1) + Ke^{-r(T-t)}N(-d_2) \end{aligned}$$

- From the above expression, one can deduce the proportions α_t and $(1 - \alpha_t)$ that have to be invested in the underlying asset and in the risk-free asset, respectively
  - $\alpha_t = \frac{S(t)N(d_1)}{S(t)N(d_1)+Ke^{-r(T-t)}N(-d_2)}$
  - $1 - \alpha_t = \frac{Ke^{-r(T-t)}N(-d_2)}{S(t)N(d_1)+Ke^{-r(T-t)}N(-d_2)}$

Managing the portfolio

- The positions on both the underlying asset and the risk-free asset must be continuously rebalanced:
  - The price of the underlying asset fluctuates
  - Times passes, which imples adjustments
    - In the proportion held in the risk-free asset
    - In the proportion held in the underlying asset due to changes in the delta value

Example

- S=100, K=100, T=1, vol = 37%, r=3%
  - P = 13 and Δ = -0.6
  - $\alpha_0 = {}^{60}/_{113} = 53\%$ - invested in stock
  - $1 - \alpha_0 = 47\%$ - invested in the risk-free asset
- Case # 1 – After 1 month, S=110
  - P = 9 and Δ = -0.64
  - $\alpha_0 = {}^{110\times0.64}/_{110\times0.64+9} = 59\%$ - invested in stock
  - $1 - \alpha_0 = 41\%$ - invested in the risk-free asset
- Case # 1 – After 1 month, S=90
  - P = 17 and Δ = -0.56
  - $\alpha_0 = {}^{90*0.56}/_{107} = 47\%$ - invested in stock
  - $1 - \alpha_0 = 53\%$ - invested in the risk-free asset

Implications of OBPI

- As the value of the original portfolio increases
  - the delta of the put becomes less negative
  - risk-free assets are sold and the position in the stock is increased (the proportion of the portfolio sold must be decreased, i.e. some of the original portfolio must be repurchased)
  - the maximum limit will be reached when the entire original portfolio has been repurchased ($\Delta_P = 0$), that is, when the portfolio is made up of only risky assets.
- As the value of the original portfolio declines
  - the delta of the put becomes more negative
  - the position in the stock portfolio is decreased (i.e. some of the original portfolio must be sold) and risk-free assets are purchased
  - the maximum limit will be reached when the entire original portfolio has been sold ($\Delta_P = -1$), that is, when the portfolio is made up of only risk free asset

Constant Proportion Portfolio Insurance

- Black and Jones introduced this method in 1986 in a research paper from the Goldman Sachs Bank with the express goal of simplifying portfolio insurance
- This strategy is also called the « cushion method »
- This strategy is very flexible
  - it can continue indefinitely as it has no time-horizon
  - the floor can easily be modified as time goes by or after significant increases

Constant Proportion Portfolio Insurance

- We can envision the portfolio according to two viewpoints
  - the portfolio is split between a risky position (exposure), S_t, and a risk-free position, M_t

$$V_t = S_t + M_t$$

  - the portfolio is the sum of the guaranteed floor, F, and the cushion, C_t :

$$V_t = F + C_t$$

- The investor must choose the initial value of the floor (that can be updated over time)

Constant Proportion Portfolio Insurance

- F = floor
- C = cushion
- V = F + C

- S = exposure
- M = risk-free position
- V = S + M

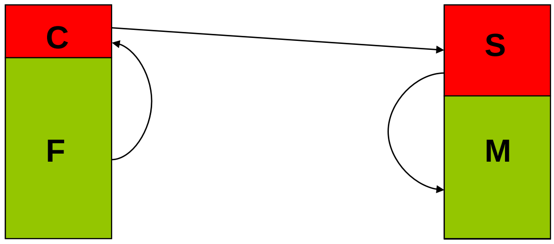
Constant Proportion Portfolio Insurance

- The multiple, mt, is defined as the ratio of the exposure to the cushion
- mt = St / Ct
- The inverse of the multiple is called management ratio, λt
- λt = 1 / mt = Ct / St
- The investor must initially choose the value of the target multiple m* (or, the value of the target management ratio λ*)

Constant Proportion Portfolio Insurance

- The strategy consists in
  - keeping a constant proportion of exposure to the risky asset
  - by keeping the amount invested in the risky asset proportional to the value of the cushion (by a factor m)
- At any time, the amount invested in the risky asset should be equal to
- St = mt × Ct

Constant Proportion Portfolio Insurance



- Step 1 : choose the floor and the multiple
- Step 2 : calculate the size of cushion (C = V – F)
- Step 3 : calculate the exposure (S = m x C)
- Step 4 : calculate the amount invested in the risk-free asset (M = V – S)
- Step 5 : dynamically rebalance

Constant Proportion Portfolio Insurance

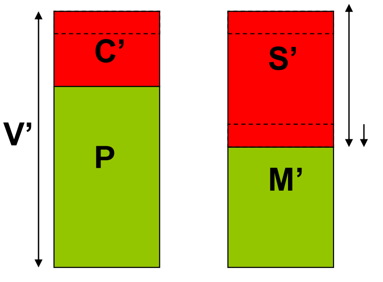
- A simple example
  - V0 = 1000
  - F is set to 900
  - initially, the cushion is C0 = 1000 – 900 = 100
  - consider a desired multiple set to m* = 4 (λ* = 0,25)
    - the amount that should be invested in the risky asset is
    - S0 = C0 × m* = 400
    - thus, by substracting this value to the portfolio value, we obtain
    - M0 = V0 – S0 = 600

Constant Proportion Portfolio Insurance

- Assume that the risky asset increase by 10%
  - S1 = 440
  - the cushion is now worth C1 = 1040 – 900 = 140
  - the multiple becomes m1 = 440 /140 = 3,14
- In order to bring m back to its target value 4, we must adjust the risky position
  - the objective is
  - S1* = 140 × 4 = 560
  - we must buy some risky asset for an amount of 120
  - conversely, the risk-free position must decrease by 120 (the strategy is self-financing)

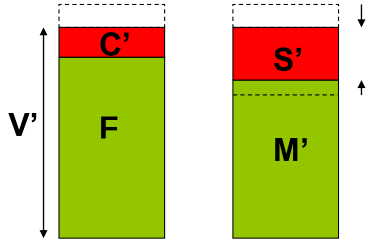
Constant Proportion Portfolio Insurance

- After an increase in the value of the risky asset
  - the portfolio value (V') increases
  - since the floor (F) is constant, the cushion (C') increases
  - the exposure (S') should be increased and the amount invested in the risk-free asset (M') should be decreased



Constant Proportion Portfolio Insurance

- After a decrease in the value of the risky asset
  - the portfolio value (V') decreases
  - since the floor (F) is constant, the cushion (C') decreases
  - the exposure (S') should be adjusted by selling some shares and the amount invested in the risk-free asset (M') should be increased



Constant Proportion Portfolio Insurance

- For a systematically increasing risky asset
  - the cushion is always increasing
  - for a €1 increase in the cushion, the exposure increases by €m
  - the amount invested in the risky-free asset tends to become negative
- For a systematically decreasing risky asset
  - the cushion tends to 0
  - the exposure tends to 0
  - the amount invested in the risk-free asset tends to the present value of the floor

Constant Proportion Portfolio Insurance

- Even for a continuous decrease in S
  - the portfolio value always remains above the floor
  - because the exposure is continuously adjusted (so that it remains proportional to the cushion)
- For a discrete decrease in S
  - the portfolio value breaks the floor if the decrease is higher than the cushion
  - the negative return the portfolio can absorb between two rebalancements is equal to the management ratio

Constant Proportion Portfolio Insurance

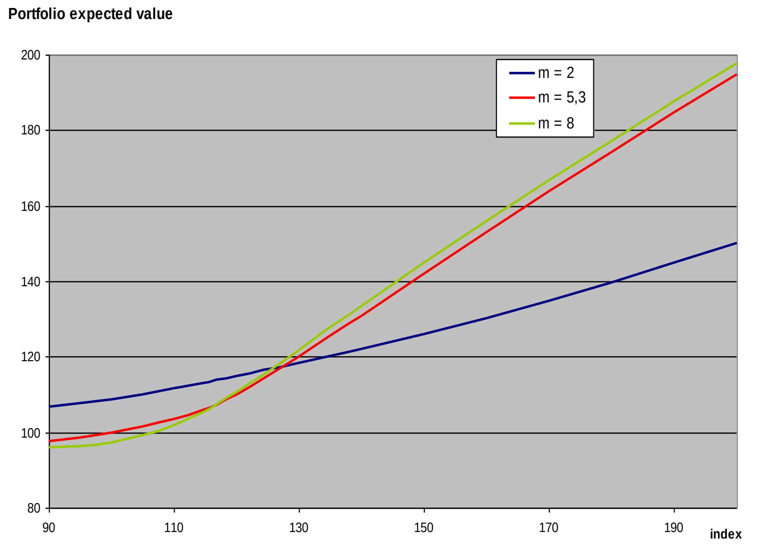
- Rebalancements may be costly in volatile markets
- Conventionally, a third parameter, the tolerance, is chosen to limit the number of portfolio adjustments
  - the portfolio is not rebalanced unless the observed multiple moves by a given percentage (20%, for example)
  - when the portfolio is rebalanced, adjustments should be such that the multiple is brought back to its target value
- Higher tolerance leads to higher gap risk
  - the strategy can guarantee the floor only for a negative return inferior to the management ratio minus its tolerance

Constant Proportion Portfolio Insurance

- In the previous example
  - the floor is broken for a market fall of 25% or more before rebalancing
- With a tolerance of 20%
  - the floor may be broken for a market fall of 1/4,8 = 20,83%
    - after an initial decrease of 21, the portfolio is
      - C1 = 79, S1 = 379, m1 = 4,797 and λ1 = 0,2084
    - it should no be rebalanced
      - now, a further decrease of 79, or 79/379 = 20,83% is enough to exhaust the cushion and break the floor

Constant Proportion Portfolio Insurance

- A higher multiple results in
  - a higher exposure
  - a portfolio that benefits better from market rises
  - a portfolio that comes closer to the floor more rapidly in bear markets
- It translates into a lower management ratio
  - in the case of discrete rebalancements the probability to break the floor (gap risk) is higher



Constant Proportion Portfolio Insurance

- momemtum strategies
  - it is necessary to have this kind of buy high/sell low scheme to obtain convex payoffs but may be seen as contrary to basic investment theory
  - some argue that such schemes tend to accentuate markets tendencies or even cause market crashes
- transaction costs
  - lead to returns that are negatively related to volatility
  - make it necessary to have some tolerance in the rebalancing scheme
    - the effective performance is modified (replication error)
    - the gap-risk increases
  - may be reduced by the use of futures
    - margin calls must be handled
    - basis risk must be accounted for

Some characteristics of PI strategies

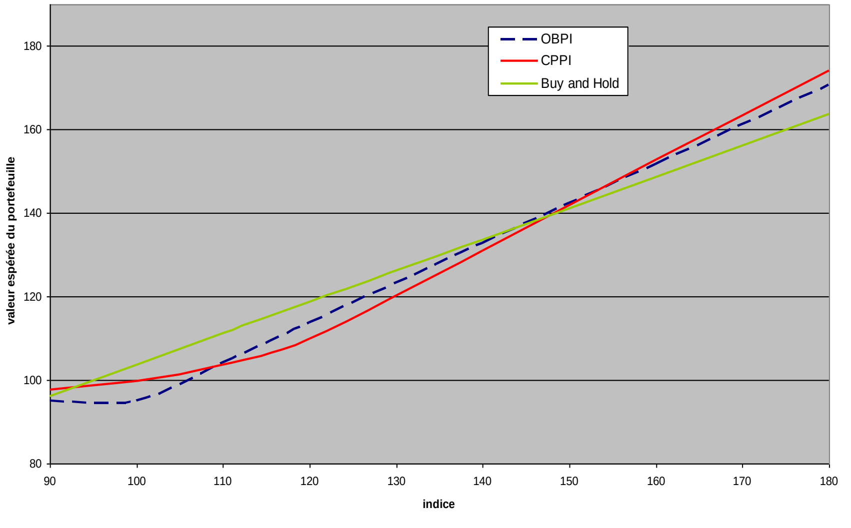
- Stochastic interest rates may cause a premature cash-out for CPPI
  - a sharp decrease in S leads to wholly invest the portfolio in safe assets
  - guarantee may not be achived in the case of a subsequent decrease in interest rates
- Gap risk
  - theoretically, portfolios must be rebalanced continuously which is not the case in reality
  - OBPI may break the floor at intermediairy dates
- Results are path-dependant
  - due to discrete rebalancing (OBPI)
  - due to the adjustement rule per se (CPPI)

Comparing the methods

- Path dependancy
  - CPPI results are more path dependant than dynamic OBPI results
  - static OBPI is path independant
- Volatility
  - P&L of static OBPI only depends on implied volatility
  - volatility estimation is critical in the case of dynamic OBPI
  - CPPI results depends on realized volatility

Comparing the methods

- Performance
  - under no arbitrage, no strategy should dominate the others in all states of nature
  - it is not straightforward to compare the performance of the different strategies
    - numerous comparison criteria can be used (expected returns, volatility of returns, gap-risk, conditionnal returns, semi-variance, …)
    - only comparable strategies should be compared (with at least the same floor and expected return)
  - CPPI tends to outperform OBPI in markets with marked tendancies (either bull or bear)



Portfolio Insurance and the crash of 1987

- Some argue that the crash was exacerbated (or even caused) by the amount of PI
  - « Investors using these strategies tend to eat like chickens and defecate like dinosaurs. They are successful for a while, and because of their success they get bigger, perhaps with the aid of leverage. Eventually, however, they lose and lose big. When that happens, they have the potential to take other investors with them, including innocent bystanders. » Bruce Jacobs, 2000.
- However, it may be that it is the underestimation of PI rather the PI itself that may cause such bubbles (Jacklin, Kleidon and Pfleiderer; 1992)