

# Event studies: principles, design, tests and some limitations

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## Outline

- 1 Introduction
- 2 Short-horizon event studies
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  - Return-generating models
  - Parameter estimates
  - Abnormal returns and their statistical properties
  - Violation of standard hypotheses
  - Analysis of power and conclusion
- 3 Long Horizon event studies
  - Introduction
  - Abnormal performance measurement

## References

### 1 Surveys

- Campbell J., Lo A. and A.C. MacKinlay (1997), *The Econometrics of Financial Markets*, Princeton University Press
- Kothari S. and J. Warner, *The Econometrics of Event Studies*, *Handbook of Corporate Finance*, B. Eckbo (editor), Elsevier-North Holland

### 2 Short-horizon event studies

- Boehmer E., Musumeci J. and A. Poulsen (1991), *Event-study Methodology Under Conditions of Event-Induced Variance*, *Journal of Financial Economics*, 30, 253–272
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- Pattel J. (1976), *Corporate Forecasts of Earnings Per Share and Stock Price Behavior: Empirical Tests*, *Journal of Accounting Research*, 14, 246–276

### 3 Long-horizon event studies

- Fama E. (1998), Market Efficiency, Long-Term Returns and Behavioral Finance, Journal of Financial Economics, 49, 283–306
- Lyon J., Barber B. and C. Tsai (1999), Improved Methods of Tests of Long-Horizon Abnormal Stock Returns, Journal of Finance, 54, 165–201.
- Mitchell M. and E. Stafford (2000), Managerial Decisions and Long-Term Stock Price Performance, Journal of Business, 73, 287–329

### Event studies: what for?

Event studies aim at quantifying the effects of an (unexpected) economic event on the value of firms

- Financial economics: corporate events, market efficiency
- Macroeconomic policy: fed rates, trade deficits
- Accounting: earning announcements
- Law and economics: changes in legal environment, and regulation – SOX, insider trading rules
- Marketing: brand strategy announcements
- ...

## Scope of the seminar

In this seminar, we will focus on how **asset prices** react to a given event

For a variety of reasons, the methodology of event-study was initiated and has been further developed in the field of financial economics

- events are reflected in asset prices (assuming markets are informationnally efficient)
- prices are easily observed
- well-performing models are available to isolate the impact of a given event on asset prices

## Pioneering works in event studies

- J. Dolley (1933): sample of 95 splits from 1921 to 1931. Prices increase in 57 of the cases and prices decline in only 26 instances
- 1930's - 1960's: increased level of sophistication  
Correction for general stock price movements and confounding events (Myers J. and A. Bakay, 1948; A. Barker, 1956,1957,1958; J. Ashley, 1962)
- Ball R. and P. Brown (1968): information content of earnings announcements
- **Fama E, Fisher L, Jensen M. and R. Roll (1969)**: effect of stock splits (after removing the effect of simultaneous dividend increases)

Since the seminal work by Fama et al., the basic layout of event studies has not changed over time: the key focus is still on measuring the sample securities' mean and cumulative mean abnormal return around the time of an event

Main changes include:

- Use of daily (and sometimes intradaily) rather than monthly security return
- The methods used to estimate abnormal returns and calibrate their statistical significance have become more sophisticated
  - new findings in the properties of long-horizon security returns
  - developments in the asset pricing literature

The making of an event studies involves 7 steps (Campbell J, Lo A. and A. MacKinlay)

- 1 Definition of the event
- 2 Selection criteria
- 3 Normal and abnormal returns
- 4 Estimation procedure
- 5 Test procedure
- 6 Results
- 7 Interpretation and conclusions

## Step 1: Definition of the event

- Definition of the event of interest
- Identification of the period over which the security prices of the firms involved in the event will be examined: the **event window**
- The analysis will be conducted in relative time: **date 0** corresponds to the **event date**
- It is customary to define an **event window** that is larger than the exact period of interest
  - Inclusion of the days prior to the event ( $-1, -2, \dots$ ) aims at accounting for possible anticipation of the event as well as information leakage
  - Inclusion of the days after the event ( $+1, +2, \dots$ ) aims at capturing posterior abnormal movements that occur after market close

## Step 2: Selection criteria

- Which firms have to be included in the study?
- Restrictions imposed by data availability and reliability
- Restrictions imposed by representativeness issues
- Some **summary statistics** (market capitalisation, average return, industry representation, distribution of events through time...) might prove useful to identify potential biases in the initial sample as well as **outliers**

## Step 3: Normal and abnormal returns

- The event of interest is expected to induce specific price movements for the sample firms
- The problem: how to isolate those movements from contemporaneous movements in a given stock that are unrelated to the event?
- The isolation of the movements that are specific to the event is made in two steps
  - 1 Computation of **abnormal returns (AR)** for each firm: **time-series** of abnormal returns for each firm
  - 2 Abnormal returns are then averaged in the **cross-section**: time series of **average abnormal returns (AAR)**
- Rationale for step 2: all selected firms experience the same event, therefore abnormal returns unrelated to the event should vanish through aggregation

## Step 3: Normal and abnormal returns

Date	Firm 1	Firm 2	...	Firm $i$	...	Firm $n$	Sample
-2	$AR_{1,-2}$	$AR_{2,-2}$	...	$AR_{i,-2}$	...	$AR_{n,-2}$	$AAR_{-2}$
-1	$AR_{1,-1}$	$AR_{2,-1}$	...	$AR_{i,-1}$	...	$AR_{n,-1}$	$AAR_{-1}$
0	$AR_{1,0}$	$AR_{2,0}$	...	$AR_{i,0}$	...	$AR_{n,0}$	$AAR_0$
+1	$AR_{1,+1}$	$AR_{2,+1}$	...	$AR_{i,+1}$	...	$AR_{n,+1}$	$AAR_{+1}$
+2	$AR_{1,+2}$	$AR_{2,+2}$	...	$AR_{i,+2}$	...	$AR_{n,+2}$	$AAR_{+2}$
+3	$AR_{1,+3}$	$AR_{2,+3}$	...	$AR_{i,+3}$	...	$AR_{n,+3}$	$AAR_{+3}$
+4	$AR_{1,+4}$	$AR_{2,+4}$	...	$AR_{i,+4}$	...	$AR_{n,+4}$	$AAR_{+4}$
+5	$AR_{1,+5}$	$AR_{2,+5}$	...	$AR_{i,+5}$	...	$AR_{n,+5}$	$AAR_{+5}$

## Step 3: Normal and abnormal returns

- **Abnormal returns** are computed as the difference between **observed returns** and **normal returns**
- Normal returns correspond to the expected returns if the event had not taken place
- Computation of abnormal returns

$$AR_{i,\tau} = R_{i,\tau}^* - E(R_{i,\tau} | X_\tau)$$

- A model is needed that permits computation (generation) of normal returns
- There exist various solutions for the computation of normal returns

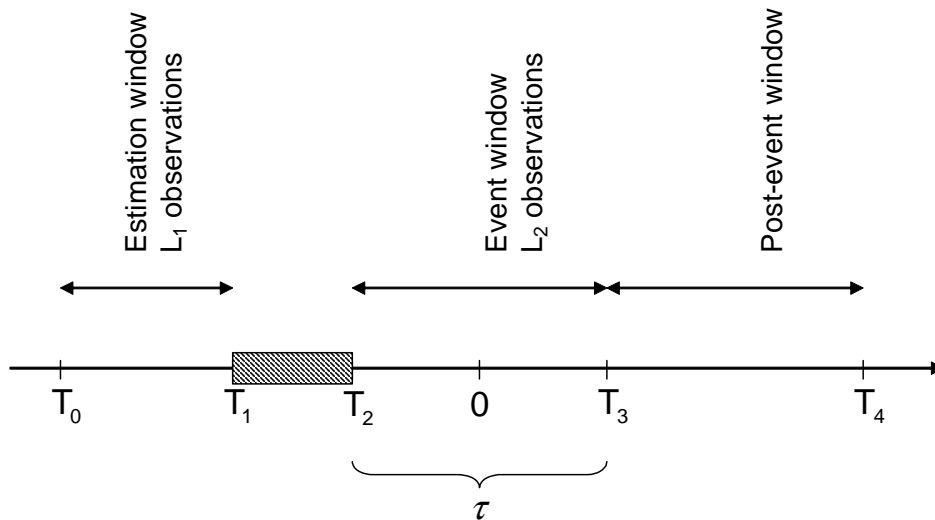
## Step 4: Estimation procedure

- The parameters of the model that is used to generate normal returns over the event window have to be **estimated**
- The estimation is performed on the **estimation window**
- It is necessary to check that the estimation window is not contaminated by events that are likely to impact the parameters of the model that generates normal returns
- Intuitively, the event window (or part of it) should not be included in the estimation window (when feasible)
- It is quite common to introduce a buffer zone between the estimation window and the event window
- Common choices for the length of the estimation window are 120 days or 250 days



## Step 4: Estimation procedure

Time line for an event study (Source: Campbell, Lo and MacKinlay)



## Step 5: Test procedure

Once abnormal returns are computed, the objective is to **test their significance**

Test procedure involves 2 steps

- 1 Step 1: aggregation of abnormal returns
  - Abnormal returns are first aggregated in the **cross section** of sample stocks to compute **AARs** on each event date
  - Average abnormal returns can also be aggregated in time-series to compute **Cumulated Average Abnormal Returns (CAR)** over the event window
  - CAR is useful to estimate the incidence of the event for claimholders' wealth

## Step 5: Test procedure

### 2 Step 2: test of the null hypothesis

- Comparison of the distributions of actual with the distribution of predicted returns
- Typically, the specific null hypothesis to be tested is whether the **mean abnormal return at time  $\tau$  is equal to 0**
- Occasionally, other parameters of the cross-sectional variation in abnormal returns can be used
  - median
  - variance
- To perform the test, one has to know the **sampling distribution** of AAR or CAR

## Step 6: Results

The empiricist has to question the reliability of his results

- Robustness tests using various sub-samples
- Incidence of outliers?
- Sensitivity to the choice of the estimation window?
- Sensitivity to the normal-return generating model?

## Step 7: Interpretation and conclusion

Either results are significant or not

- What do they mean?
- Are they different from those obtained in other comparable studies? Why?
- Differences in subsamples? Why?
- Sensitivity to outliers? Are outliers a proxy for some unidentified factor?
- ...

Various solutions exist. We will focus on two of them

- Constant mean-return model
- Market model

Both models are both easy to implement and reliable in detecting abnormal returns

## The constant mean-return model

### Definition and assumptions

- The constant mean-return model posits that firm  $i$  return on date  $t$  is given by

$$R_{i,t} = \mu_i + \varepsilon_{i,t}$$

- $\mu_i$  is firm  $i$  average return
- $\varepsilon_{i,t}$  is the innovation on date  $t$
- The following (simplifying) assumptions are made:
  - $E(\varepsilon_i) = 0$
  - $\sigma^2(\varepsilon_{i,t}) = \sigma^2(\varepsilon_i), \forall t$  (homoscedasticity)
  - $E(\varepsilon_{i,t}\varepsilon_{i,t'}) = 0, \forall t \neq t'$  (random walk - market efficiency)

## The market model (Sharpe, 1963)

### Definition and assumptions

- The market model posits a linear relationship between  $R_{i,t}$  and the market return on the same date, denoted  $R_{m,t}$  (proxied by the return of an appropriate stock index - eg S&P 500)

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \xi_{i,t}$$

- $\alpha_i$  and  $\beta_i$  are the two parameters of the market model for firm  $i$
- $\xi_{i,t}$  is a disturbance term
  - $E(\xi_i) = 0$
  - $\sigma^2(\xi_{i,t}) = \sigma^2(\xi_i), \forall t$  (homoscedasticity)
  - $E(\xi_{i,t}\xi_{i,t'}) = 0, \forall t \neq t'$

## Other return-generating models

### Statistical models

- **Index model** – Equivalent to the market model where  $\alpha_i$  is constrained to 0 and  $\beta_i$  is constrained to 1:  $R_{i,t} = R_{m,t}$
- **Factor models** – The goal is to reduce the variance of the abnormal returns by explaining more of the variation in the normal return. Common choices include:

- The **Fama-French** 3-factor model

$$R_{i,t} = \alpha_i + \beta_{1,i}R_{m,t} + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \eta_{i,t}$$

- The **Carhart** 4-factor model

$$R_{i,t} = \alpha_i + \beta_{1,i}R_{m,t} + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \beta_{4,i}UMD_t + \zeta_{i,t}$$

## Other return-generating models

### Economic models

- Economic models restrict the parameters of statistical models to provide more constrained return-generating models. The **CAPM** and the **APT** are (used to be) common choices
- The CAPM was commonly used in event studies in the 70s. Since then, deviations from the CAPM have been discovered. This casts doubts on the validity of the underlying restrictions: the market model is now preferred
- Estimation of factors makes the use of APT complicated. The APT offers no clear advantage compared with an unrestricted statistical model

- The parameters of the various statistical models – constant-mean, market, Fama-French, Carhart – must be **estimated** on the **estimation window**
- It is possible to estimate the parameter(s) of any of these models using a matrix representation and applying the appropriate specification so as to estimate the set of parameters of the selected return-generating model

## Estimation: matrix representation of data

- Firm  $i$  returns on the estimation window can be written as:

$$\mathbf{R}_i = \mathbf{X}_i \theta_i + \epsilon_i \quad (1)$$

where  $\mathbf{R}_i = [R_{i,T_0}, \dots, R_{i,T_1}]'$  is the vector of firm  $i$   $L_1$  returns on the estimation window  $[T_0; T_1]$

- $\mathbf{X}_i$  reduces to a  $L_1$ -row column vector of ones for the **constant-mean model**
- $\mathbf{X}_i$  is a  $L_1 \times 2$  matrix with ones in the firm column and the  $L_1$  market returns on dates  $T_0$  to  $T_1$  for the **market model**
- $\theta_i$  corresponds to the vector of parameters associated with the model of interest
  - $\theta_i$  reduces to the scalar  $\mu_i$  for the **constant-mean model**
  - $\theta_i$  is equal to vector  $[\alpha_i, \beta_i]'$  for the **market model**

## Estimation: matrix representation of data

- Irrespective of the underlying model, the estimation of the parameters can be made through OLS and the following results obtain

$$\hat{\theta}_i = (\mathbf{X}'_i \mathbf{X}_i)^{-1} \mathbf{X}'_i \mathbf{R}_i \quad (2)$$

$$\hat{\sigma}^2(\epsilon_i) = \frac{1}{L_1 - k} \hat{\epsilon}'_i \hat{\epsilon}_i \quad (3)$$

$$\hat{\epsilon}_i = \mathbf{R}_i - \mathbf{X}_i \hat{\theta}_i \quad (4)$$

$$\sigma^2(\hat{\theta}_i) = (\mathbf{X}'_i \mathbf{X}_i)^{-1} \sigma^2(\epsilon_i) \quad (5)$$

where  $k = 1$  for the constant-mean model and  $k = 2$  for the market model

- Abnormal returns** are computed for each firm on each date  $[T_2; T_3]$  of the event window as the difference between observed returns and predicted returns
- Using the previously defined notations:

$$\hat{\epsilon}_i^* = \mathbf{R}_i^* - \mathbf{X}_i^* \hat{\theta}_i$$

where:

- $\mathbf{R}_i^*$  is the  $L_2$  -row vector of observed returns on the event date
- $\mathbf{X}_i^*$  is either a  $L_2$ -row vector of ones or a  $L_2 \times 2$  matrix with ones on the first column and market returns on dates  $T_2$  to  $T_3$  on the second column
- $\hat{\theta}_i$  is the vector of parameter estimates

## The case of a single asset

- The abnormal returns for firm  $i$  on dates  $[T_2; T_3]$  will be distributed with zero mean and a  $L_2 \times L_2$  covariance matrix whose value is given by:

$$V_i = \sigma^2(\epsilon_i)\mathbf{I} + \sigma^2(\epsilon_i)[\mathbf{X}_i^*(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i^{*'}] \quad (6)$$

where  $\mathbf{I}$  is the  $L_2 \times L_2$  identity matrix

- The first term in (6) is the variance due to future disturbances whereas the second term is the additional variance due to sampling error in  $\hat{\theta}_i$
- Notice that the second term goes to 0 when the length  $L_1$  of the estimation window becomes large
- Most tests assume that  $V_i$  reduces to  $\sigma^2(\epsilon_i)\mathbf{I}$ : the variance of the abnormal returns is thus  $\sigma^2(\epsilon_i)$

## The case of a single asset

- For the market model, (6) reduces to

$$Q_{i,\tau} = \sigma^2(\epsilon_i) \underbrace{\left\{ 1 + \frac{1}{L_1} + \frac{(R_{m,\tau} - \bar{R}_m)^2}{\sum_{t=T_0}^{T_1} (R_{m,t} - \bar{R}_m)^2} \right\}}_{C_{i,\tau}}$$

- $C_i$  reflects the increase in variance due to prediction outside the estimation window (estimation is made out-of-sample)
- Notice that  $C_i$  goes to 1 when  $L_1$  goes to  $+\infty$



## The case of a single asset

- To test whether  $\hat{\epsilon}_{i,\tau}^*$  is different from 0 on a given date, we need the **additional restrictive assumption** that the (abnormal) return distributions are **normal**
- Under this assumption, the following results obtain:

$$\frac{\hat{\epsilon}_{i,\tau}^*}{\sigma(\epsilon_i)} \sim N(0, 1) \text{ or } \frac{\hat{\epsilon}_{i,\tau}^*}{\sigma(\epsilon_i)\sqrt{C_{i,\tau}}} \sim N(0, 1) \quad (7)$$

$$\frac{(L_1 - k)\hat{\sigma}^2(\epsilon_i)}{\sigma^2(\epsilon_i)} \sim \chi^2(L_1 - k) \quad (8)$$

$$\frac{\hat{\epsilon}_{i,\tau}^*}{\hat{\sigma}(\epsilon_i)} \sim t(L_1 - k) \text{ or } \frac{\hat{\epsilon}_{i,\tau}^*}{\hat{\sigma}(\epsilon_i)\sqrt{C_{i,\tau}}} \sim t(L_1 - k) \quad (9)$$

## Cross-sectional aggregation

### Brown and Warner (1980) – Constant-mean return

- As cross-sectional independence may not hold in the constant-mean return model, one way to deal with this issue is to form equally-weighted portfolios over both the estimation window and the event window
- On a given event date, the abnormal return for the portfolio, assuming N event firms, is computed just as

$$AAR_\tau = \frac{1}{N} \sum_{i=1}^N \hat{\epsilon}_{i,\tau}^* \quad (10)$$

## Cross-sectional aggregation

### Brown and Warner (1980) – Constant-mean return (cont'd)

- The variance of the AARs on the estimation window can be estimated as

$$\frac{1}{L_1 - 1} \sum_{t=T_0}^{T_1} (AAR_t - \overline{AAR})^2 \quad (11)$$

where

$$\overline{AAR} = \frac{1}{NL_1} \sum_{t=T_0}^{T_1} \sum_{i=1}^N AR_{i,t}$$

## Cross-sectional aggregation

### Brown and Warner (1980) – Constant-mean return (cont'd)

The statistic

$$\frac{\frac{1}{N} \sum_{i=1}^N AR_{i,\tau}}{\sqrt{\frac{1}{L_1 - 1} \sum_{t=T_0}^{T_1} (AAR_t - \overline{AAR})^2}}$$

is Student-t with  $(L_1 - 1)$  degrees of freedom

## Cross-sectional aggregation

### Brown and Warner (1980) – Market model

- Under the market model, residual cross-correlation is likely to be small
- $AAR_{\tau}$  on the event window are calculated in the same way as in (10)
- Since we assume cross-sectional independence, the standard deviation of the  $AAR_{\tau}$  is given by

$$\frac{1}{N} \sqrt{\sum_{i=1}^N \sigma_i^2(\epsilon_i)} \quad (12)$$

## Cross-sectional aggregation

### Brown and Warner (1980) – Market model (cont'd)

- Since, (12) is unknown, it must be estimated. An unbiased estimator is:

$$\frac{1}{N} \sqrt{\sum_{i=1}^N \hat{\sigma}_i^2(\epsilon_i)}$$

- Thus, the statistic

$$\frac{\sum_{i=1}^N \hat{\epsilon}_{i,\tau}^*}{\sqrt{\sum_{i=1}^N \hat{\sigma}_i^2(\epsilon_i)}}$$

is Student-t with  $(L_1 - 2)$  degrees of freedom

## Cross-sectional aggregation

### Pattel (1976) – Market model

- In Pattel (1976), residuals are standardized according to (7) – 2nd version
- First, standardization adjusts for the fact that the event-period residual is an out-of-sample prediction and hence, it will have a higher standard deviation than estimation-window residuals
- Second, standardizing the event-period **before forming portfolios** allows to prevent securities with large variances from dominating the test

## Cross-sectional aggregation

### Pattel (1976) – Market model

- Contrary to the previous statistics, the approach does not rely on  $AAR_\tau$  as defined in (10) but on standardized residuals
- The average of standardized residuals is clearly equal to 0
- Since  $\frac{\hat{\epsilon}_{i,\tau}^*}{\hat{\sigma}(\epsilon_i)\sqrt{C_{i,\tau}}} \sim t(L_{i,1} - 2)$ , its variance is  $(L_{i,1} - 2)/(L_{i,1} - 4)$
- The average variance of the standardized residuals is thus equal to  $1/N \sum_{i=1}^N (L_{i,1} - 2)/(L_{i,1} - 4)$

## Cross-sectional aggregation

### Pattel (1976) – Market model

- Assuming  $N$  is large ( $N \geq 30$ ) and cross-sectional independence, we can apply the Lindberg-Levy version of the central-limit theorem (with unequal variances), so that:

$$\frac{\frac{\sqrt{N}}{N} \sum_{i=1}^N \frac{\hat{\epsilon}_{i,\tau}^*}{\hat{\sigma}(\epsilon_i) \sqrt{C_{i,\tau}}}}{\sqrt{\frac{1}{N} \sum_{i=1}^N \frac{L_{i,1} - 2}{L_{i,1} - 4}}} = \frac{\sum_{i=1}^N \frac{\hat{\epsilon}_{i,\tau}^*}{\hat{\sigma}(\epsilon_i) \sqrt{C_{i,\tau}}}}{\sqrt{\sum_{i=1}^N \frac{L_{i,1} - 2}{L_{i,1} - 4}}} \rightarrow N(0, 1) \quad (13)$$

## Times-series aggregation – CAR

We first consider the case of an individual asset

- Using (7),  $\frac{1}{\sqrt{L_2}} \sum_{\tau} \frac{\hat{\epsilon}_{i,\tau}^*}{\sigma(\epsilon_i) \sqrt{C_{i,\tau}}} \sim N(0, 1)$

- Thus:

$$\sum_{\tau} \frac{\hat{\epsilon}_{i,\tau}^*}{\hat{\sigma}(\epsilon_i) \sqrt{L_2 C_{i,\tau}}} \sim t(L_1 - 2) \quad (14)$$

## Times-series aggregation – CAR

### Brown and Warner (1980) – Constant-mean return model

- Let denote  $CAAR = \sum_{\tau} AAR_{\tau}$
- Clearly,  $E(CAAR) = 0$ , and assuming time-series independence,  $\sigma(ACAR) = \sqrt{L_2}\sigma(AAR_{\tau})$
- The unbiased estimator  $\hat{\sigma}^2(AAR_{\tau})$  is the same as in (11), so that

$$\frac{\sum_{\tau} AAR_{\tau}}{\sqrt{\frac{L_2}{L_1 - 1} \sum_{t=T_0}^{T_1} (AAR_t - \overline{AAR})^2}}$$

is Student-t with  $(L_1 - 1)$  degrees of freedom

## Times-series aggregation – CAR

### Brown and Warner (1980) – Market model

- Let  $CAR_i = \sum_{\tau} \hat{\epsilon}_{i,\tau}^*$
- $E(CAR_i) = 0$ , and assuming time-series independence,  $\sigma(CAR_i) = \sqrt{L_2}\sigma(\epsilon_i)$
- Let  $ACAR = 1/N \sum_{i=1}^N CAR_i$
- $E(ACAR) = 0$ , and assuming cross-sectional independence,  $\sigma(ACAR) = 1/N \sqrt{\sum_{i=1}^N \sigma^2(CAR_i)}$ , which is equal to  $\sqrt{L_2}/N \sqrt{\sum_{i=1}^N \sigma^2(\epsilon_i)}$

## Times-series aggregation – CAR

### Brown and Warner (1980) – Market model (cont'd)

- Since  $\sigma^2(\epsilon_i)$  is unknown, we must use the unbiased estimator  $\hat{\sigma}_i^2(\epsilon_i)$
- Thus the statistic

$$\frac{\sum_{i=1}^N \sum_{\tau} \hat{\epsilon}_{i,\tau}^*}{\sqrt{L_2} \sqrt{\sum_{i=1}^N \hat{\sigma}_i^2(\epsilon_i)}}$$

is Student-t with  $(L_1 - 2)$  degrees of freedom

## Times-series aggregation – CAR

### Pattel (1976) – Market model

- From (14), we know that the distribution of (standardized)  $CAR_i$  is Student-t with  $(L_{i,1} - 2)$  degrees of freedom
- The variance of  $CAR_i$  is thus  $(L_{i,1} - 2)/(L_{i,1} - 4)$
- Therefore, the average variance of the  $CAR_i$  in the cross-section is  $1/N \sum_{i=1}^N (L_{i,1} - 2)/(L_{i,1} - 4)$

## Times-series aggregation – CAR

### Pattel (1976) – Market model

- Assuming  $N$  is large ( $N \geq 30$ ) and cross-sectional independence, we can apply the Lindberg-Levy version of the central-limit theorem (with unequal variances), so that:

$$\frac{\sum_{i=1}^N \sum_{\tau} \frac{\hat{\epsilon}_{i,\tau}^*}{\hat{\sigma}(\epsilon_i) \sqrt{L_2 C_{i,\tau}}}}{\sqrt{\sum_{i=1}^N \frac{L_{i,1} - 2}{L_{i,1} - 4}}} \rightarrow N(0, 1)$$

## Non parametric tests

- So far, we have assumed that abnormal returns (residuals) are normally distributed, which allowed us to derive the sampling distribution of  $AR$ ,  $AAR$  as well as  $CAR$
- We thus performed **parametric tests**
- In **non parametric tests**, no assumption is made as to the distribution of abnormal returns



## Non parametric tests

### The sign test

- The sign test is a cross-sectional test that is based on the size of the abnormal returns (both the sample of  $AR_i$ s or  $CAR_i$ s can be used)
- It requires that the  $AR_i$ s or  $CAR_i$ s are **independent across securities** and that the **expected proportion of positive** (cumulated) abnormal returns under the **null hypothesis is .5**
- If, for example, the **alternative hypothesis** is that there is a positive abnormal return associated with the event, the null hypothesis is  $H_0 : p \leq 0.5$  and the alternative is  $H_a : p > 0.5$ , where  $p = Pr(AR \text{ or } CAR \geq 0)$

## Non parametric tests

### The sign test (cont'd)

- Under the previously defined null hypothesis, the statistic requires the computation of  $N^+$ , where  $N^+$  is the number of cases when the (cumulated) abnormal return is positive
- Asymptotically when  $N$  increases, we have:

$$\left[ \frac{N^+}{N} - 0.5 \right] \frac{\sqrt{N}}{0.5} \sim N(0, 1)$$

## Non parametric tests

### Corrado (1989) rank test

- A weakness of the sign test is that it may not be well specified if the distribution of abnormal return is **skewed**
- This is likely to happen with daily or intradaily data
- With skewed abnormal returns, the expected proportion of positive abnormal returns can differ from 0.5 even under the null hypothesis
- The Corrado (1989) rank test addresses this shortcoming

## Non parametric tests

### Corrado (1989) rank test (cont'd)

- Drawing on previous notations, consider the sample of  $(L_1 + L_2)$  returns for the  $N$  event firms
- The  $(L_1 + L_2)$  abnormal returns on each firm  $i$  are ranked from 1 to  $(L_1 + L_2)$
- Let denote  $K_{i,\tau}$  the rank of the abnormal return of firm  $i$  for the event date  $\tau$
- The rank test uses the fact that the expected rank under the null hypothesis is  $(L_1 + L_2)/2$

## Non parametric tests

### Corrado (1989) rank test (cont'd)

- The test statistics (whose asymptotic distribution is standard normal) for the null hypothesis of no abnormal return on event day  $\tau$  is computed as

$$\frac{1}{N} \sum_{i=1}^N N \left( K_{i,\tau} - \frac{L_1 + L_2}{2} \right) / S(K)$$

where  $S(K)$  is equal to

$$S(K) = \sqrt{\frac{1}{L_1 + L_2} \sum_{t=1}^{L_1+L_2} \left( \frac{1}{N} \sum_{i=1}^N \left( K_{i,t} - \frac{L_1 + L_2}{2} \right) \right)^2}$$

## Clustering

- In analyzing aggregated abnormal returns, we have thus far assumed that abnormal returns on individual securities are **uncorrelated in the cross-section**
- This is a reasonable assumption as long as the event windows of the sample securities do not **overlap in calendar time**
- In presence of overlapping, covariances between abnormal returns may differ from 0

## Clustering

- When there is a single common event date, clustering can be accommodated in two different ways
  - Abnormal returns can be aggregated in a portfolio, which allows to account for cross-correlation of the abnormal returns
  - A second way to handle clustering is to analyze the abnormal returns without (cross-sectional) aggregation
  - The vector of individual abnormal returns (including both the estimation and the event window) is then regressed against dummy variables for the event window dates
  - **heteroscedasticity** might be an issue so that White correction might prove necessary

## Event-induced variance

- So far, the test statistics we have studied rely on event-window estimates of the variance of abnormal returns
- As the event is likely to increase uncertainty, variance is likely to increase on the event window
- This will bias the tests towards a higher rejection rate of the null hypothesis that mean abnormal returns are equal to 0
- To correct this bias, we need to eliminate reliance on past returns in estimating the variance of abnormal returns
- This approach underlies the test statistic proposed by Boehmer, Musumeci and Poulsen (1991)

## Event-induced variance

- Let denote  $SR_{i,0}$  the standardized residual as defined in (9) on the event day, ie:

$$SR_{i,0} = \frac{\hat{\epsilon}_{i,0}^*}{\hat{\sigma}(\epsilon_i) \sqrt{C_{i,0}}}$$

- The Boehmer et al. statistic (*standardized cross-sectional method*), which is asymptotically standard normal is given by:

$$\frac{\frac{1}{N} \sum_{i=1}^N SR_{i,0}}{\sqrt{\frac{1}{N(N-1)} \sum_{i=1}^N \left( SR_{i,0} - \sum_{i=1}^N \frac{SR_{i,0}}{N} \right)^2}}$$

## Partially anticipated events

- When investors partially anticipate an event, the announcement resolves uncertainty concerning event's timing
- The **announcement effect** thus only reflects the change in firm value attributable to this resolution of uncertainty
- Therefore, the announcement effect underestimates the actual **economic impact** of the event (unless the event was thought impossible prior to the announcement)
- Malatesta and Thompson (1985) develop a model of stock price reactions to partially anticipated events and propose a test procedure that allows to disentangle the announcement effect and the economic effect

## Event-date uncertainty

- The usual method of handling event-date uncertainty is to expand the event window to avoid the risk of missing the event
- Ball and Torous (1988) investigate this issue and develop a maximum-likelihood procedure which accommodates event-date uncertainty
- The improvement is small so that there is little gain from their elaborate estimation framework compared with the informal procedure

## Other issues

- Under a noisy estimation period, parameter estimates are noisy too. Aktas, De Bodt and Cousin (2007) address this issue using a mixture of distributions with a 2-regime market model. The estimation is performed using the EM algorithm
- Some event dates might be endogenous: returns predict event (IPOs, acquisitions,...). Viswanathan and Wei prove that abnormal returns are negative and propose an estimation technique that allows to correct for this bias

- Using test statistics, errors are of two types
  - **Type I** error occurs when the the null hypothesis is falsely rejected
  - **Type II** error occurs when the null hypothesis is falsely accepted
- Accordingly, two key properties of event study tests have been investigated
  - The first is whether the test statistic is correctly specified. A correctly specified test yields a Type I error probability equal to the assumed size of the test (eg 1%, 5%)
  - The second concern is power, i.e. a test's ability to detect abnormal performance when it is present.
  - When comparing tests that are well specified, those with higher power are preferred

## The joint test issue

- While the specification and power of a test can be statistically determined, economic interpretation is not straightforward because all tests are joint tests
- Indeed, an event study is a test of whether abnormal returns are zero and whether the assumed return-generating model is correct

## Brown-Warner simulations

- To address the issue of event study properties, the standard tool is to employ simulation procedures that use actual security returns
- Different event study methods are simulated by repeated application of each method to samples that have been constructed through a **random selection of securities** and **random selection of an event date to each**
- If abnormal returns are measured correctly, these samples should show **no abnormal performance**, on average

This makes possible to study **test statistic specification**, i.e. rejecting the null hypothesis when it is known to be true

## Brown-Warner simulations (cont'd)

- Further, various levels of a deterministic abnormal performance (eg 0.1%, 0.5%, 1%) are artificially introduced in the sample
- This technique also allows to study the behavior of the test in presence of abnormal variance: eg Boehmer et al. introduce shocks that correspond to a random drawing from a normal distribution with a mean of 0%, 1%, 2% and a variance of  $k\sigma_i^2$  where  $k$  equals 0, 0.5, 1 and 2

This permits direct study of the **power** of the event study tests, i.e. the ability to detect a given level of abnormal performance



Table 2

Average rejection rates for various test statistics for 250 portfolios of 50 securities (from 1962 to 1986) each at several significance levels. The test statistics test the null hypothesis that the average abnormal return is zero. Abnormal performance is a random variable drawn from  $N(0, k\sigma^2)$ , where  $k$  is a constant and  $\sigma^2$  is either the individual-security or the average estimation-period variance.

	<i>Panel A</i>				<i>Panel B</i>			
	Event-induced variance proportional to individual-security estimation-period variance				Event-induced variance proportional to average estimation-period variance			
	One-tailed tests		Two-tailed tests		One-tailed tests		Two-tailed tests	
	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$
	$k = 0.5$							
Traditional <sup>a</sup>	0.128	0.052	0.152	0.064	0.124	0.064	0.136	0.056
Standardized-residual <sup>b</sup>	0.148	0.052	0.152	0.052	0.188	0.100	0.236	0.128
Sign <sup>c</sup>	0.052	0.012	0.088	0.016	0.068	0.008	0.080	0.000
Ordinary cross-sectional <sup>d</sup>	0.092	0.004	0.080	0.012	0.088	0.020	0.072	0.032
Method-of-moments <sup>e</sup>	0.068	0.036	0.100	0.036	0.100	0.028	0.100	0.044
Standardized cross-sectional <sup>f</sup>	0.084	0.020	0.068	0.028	0.092	0.028	0.088	0.028

## Conclusion on short-horizon event studies

- Specification is good
- Power is high when abnormal performance is concentrated in the event window and low otherwise
- Abnormal returns do not crucially depend on the return-generating model
- However, some models are better in addressing the issue of cross-sectional and time-series dependence as well as the variance of abnormal returns
- Power increases with sample size

All event studies, regardless of the horizon length must deal with several basic issues:

- 1 Risk adjustment and return-generating models
- 2 Aggregation of abnormal returns
- 3 Calibration of statistical significance of abnormal returns

The problem is that these issues become **critically** important with long horizons

## Error in risk adjustment

The problem of risk adjustment is exacerbated in long horizon event studies because the potential for such error is greater  
In many event studies:

- The event follows unusual prior performance (e.g. stock splits follow good performance)
- The sample consists of firms with extreme (economic) characteristics (e.g. low market capitalisation stocks, low-priced stocks...)
- The event is defined on the basis of unusual prior performance (e.g. contrarian investment strategies)

## Model for expected returns

- Fama (1998), "all models for expected returns are incomplete descriptions of the systematic patterns in average returns"
- With the rejection of the CAPM a quest for a better-and-improved model has began which resulted in the Fama-French 3-factor model and Carhart 4-factor model
- There is no sound economic rationale motivating those models (Yogo, JF-2006 ?), but they must be used since they are able to capture the cross-section of stock returns

## Significance tests

- Long-horizon returns, even after adjustment, tend to be right-skewed: standard tests are thus biased
- The skewness property seems to arise from the lack of independence arising from overlapping long-horizon return observations in event portfolios, which causes cross-correlation
  - corporate events like mergers and share repurchases exhibit waves
  - some industries might be over-represented in the event sample (e.g. merger activity among technology stocks)

Two main methods exist for assessing and calibrating post-event risk-adjusted performance:

- 1 The characteristic-based matching approach, also known as the **Buy and Hold Abnormal Return (BHAR)** approach
- 2 The Jensen's alpha approach, also known as the **calendar-time portfolio** approach

Unfortunately, both have low power and neither is immune to misspecification

## The BHAR approach

In a nutshell, BHAR returns are "the average multiyear return from a strategy of investing in all firms that complete an event and selling at the end of a prespecified holding period versus a comparable strategy using otherwise similar nonevent firms" (Mitchell and Stafford, 2000)

- The BHAR approach is interesting in that it better resembles investors' actual investment experience
- However, BHAR hinges on the validity of the assumption that event firms differ from the "otherwise similar nonevent firms" only in that they experience the event

## The BHAR approach (cont'd)

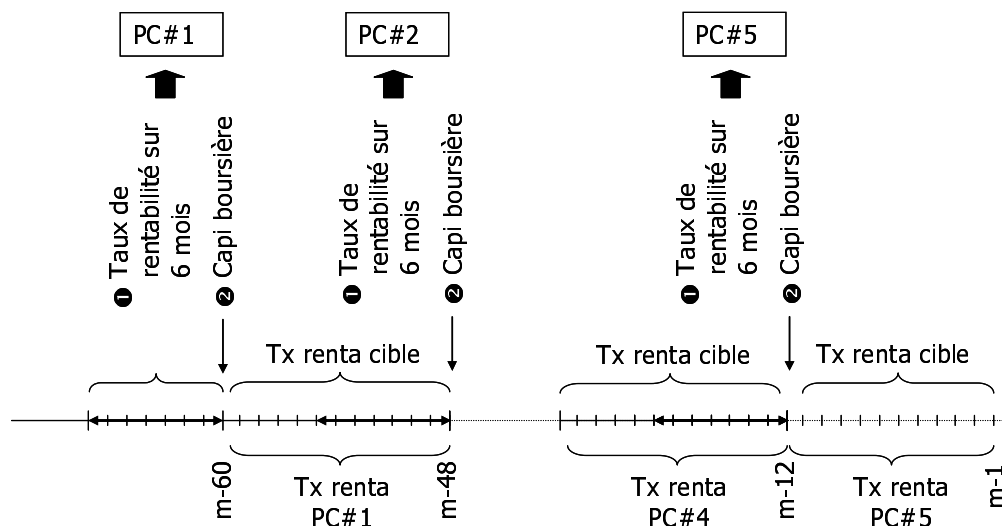
- Once a matching firm or portfolio is identified, BHAR calculation is done in the following way

$$BHAR_i(t, T) = \prod_{t=1}^T (1 + R_{i,t}) - \prod_{t=1}^T (1 + R_{B,t})$$

where  $R_{B,t}$  is the return on either a non-event firm that is matched to the event-firm  $i$ , or it is the return on a matched (benchmark) portfolio

- The matching procedure depends on the return-generating model that is thought to be an adequate description of expected returns

## Riva and Thauvron, 2002



## The Jensen-alpha approach

- Assume a sample of firms experience a corporate event (e.g. an IPO or a SEO). The event might be spread across firms over several years or decades (the sample period)
- Assume we want to estimate the price performance over 2 years (24 months) following the event for each sample firm
- In each calendar month over the entire sample period, a portfolio is constructed comprising all firms experiencing the event within the previous  $T$  months
- The portfolios are rebalanced each month and an equal or value-weighted excess return (with respect to the prevailing  $R_f$ ) is calculated

## The Jensen-alpha approach (cont'd)

- The resulting time-series of excess returns is regressed on the CAPM or the Fama-French 3 factor or the Carhart 4-factor model
- The intercept corresponds to the average monthly abnormal performance